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COMPUTATIONS OF THE RADIATION PATTERN OF A REFLECTOR ANTENNA.(U)
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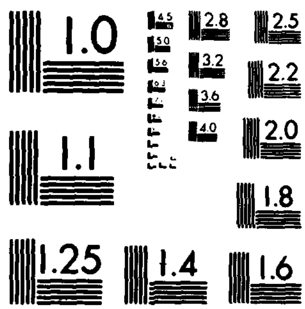


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This report describes a mathematical approach to compute the radiation pattern of a reflector antenna. The radiation integral is expanded into an orthogonal series in terms of a field point coordinate. Since this series is orthogonal, coefficients of this series can be easily determined. Once these coefficients are known, radiation patterns at any field point can be readily computed. Since this approach avoids performing the integration of the radiation integral at each field point, it greatly reduces the required computation. A brief description of the computer program is also included in this report.

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COMPUTATION OF THE RADIATION PATTERN OF A REFLECTOR ANTENNA

## INTRODUCTION

The radiation pattern of a reflector antenna is a function of the distribution of the surface current on the reflector. Once this surface current is known, the antenna pattern can be computed directly by integrating this current distribution over the reflector surface. This surface current is induced by the incident field from the primary feed source. If the primary feed pattern is known, it is straight forward to compute the incident field on the reflector surface and hence the induced surface current. This process in general involves an integration of the surface current. In the past, the primary feed is located at the focal point and its radiation pattern is known in terms of the reflector coordinate. The induced surface current on the reflector is hence readily to be written down. However, in some cases, the primary feed is not necessarily located at the focal point and its pattern may be known at a different coordinate system. It is thus necessary to have a coordinate transformation before the reflector surface current can be determined.

Another aspect of this problem is that the Green's function of this integration involves both the coordinates of the surface current and the coordinates of the field points. Each time the radiation field of a different field point is computed, this integration must be performed over again. Unfortunately this integration in general cannot be integrated into a closed form. Repeating of this numerical integration is time consuming, particularly when a large number of field points must be computed. In this report, a method is presented in which the Green's function is expanded into an orthogonal series which can be integrated into a closed form. The secondary radiation pattern is the summation of this series. Hence, it requires only to perform this integration once to determine the coefficients of the series. It thus greatly reduces the required computation time.

## RADIATION FIELD OF A REFLECTOR ANTENNA

The radiation pattern of a reflector antenna is determined by the surface current distribution on the reflector. The radiation E field is related to the surface current by the following relations:

$$E_\Theta = - j \frac{\omega\mu_o}{4\pi R} e^{-jk_oR} \iint_s \hat{\Theta} \cdot \vec{J}_s \exp (jk_o\vec{\rho}\cdot\hat{R}) d_s \tag{1a}$$

$$E_\Phi = - j \frac{\omega\mu_o}{4\pi R} e^{-jk_oR} \iint_s \hat{\Phi} \cdot \vec{J}_s \exp (jk_o\vec{\rho}\cdot\hat{R}) d_s \tag{1b}$$

Where R is the distance from the focal point (reference point) of the reflector to the far field point and $\vec{\rho}$ is a vector pointing from the reference point to a point on the reflector (Figure 1). While $\hat{\Theta}$ and $\hat{\Phi}$ are respectively the unit vector of the spherical coordinate of the reflector,

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equations (1a) and (1b) can be written

$$\vec{E}_\theta = \hat{\theta} \cdot \vec{I} \tag{2a}$$

$$\vec{E}_\phi = \hat{\phi} \cdot \vec{I} \tag{2b}$$

$$\vec{I} = -j \frac{\omega\mu_o}{4\pi R} e^{-jk_oR} \iint \vec{J}_s \exp (jk_o\vec{\rho}\cdot\hat{R})d_s \tag{2c}$$

It is thus straightforward to compute the radiation pattern of a reflector once the surface current distribution is known. This surface current is induced by the incident field which can be found from the following relation

$$\vec{J}_s = 2 (\hat{n}x\vec{H}_i) = 2\sqrt{\frac{\varepsilon_o}{\mu_o}} [\hat{n}x(\vec{\rho}'x\vec{E}_i)] \tag{3}$$

where $\hat{n}$ is the unit vector normal to the reflector surface and $\vec{\rho}'$ is a radial vector from the primary source to the reflector surface while $\vec{E}_i$ is the incident E- field.

Let us assume that the primary feed is far away from the reflector surface and the field incident on the reflected surface can be considered at a far-zone region. Furthermore, mutual coupling effect can be neglected. Under these conditions, the incident field on the reflector can be represented in the following form. [1,2]

$$\bar{E}_i = \sqrt{2} \left(\frac{\mu_o}{\varepsilon_o}\right)^{1/4} \left[\frac{P_tG_f(\theta',\phi')}{4\pi}\right]^{1/2} \frac{\exp(jk_o\rho')}{\rho'} \hat{e}_i \tag{4}$$

where $P_t$ is the total radiating power from the primary feed and $G_f(\theta', \phi')$ is the radiation pattern of the primary feed which usually is known in terms of the feed coordinator $\theta'$ and $\phi'$. The quantity $\rho'$ is the distance from primary feed to the reflector surface. Vector $\hat{e}_i$ is the unit vector which defines the direction of this incident feed on the reflected surface. If the primary feed is linearly polarized at a direction $\vec{\mu}$, this vector $\hat{e}_i$ is then in the direction $\mu$ and normal to vector $\vec{\rho}'$. This can be expressed

$$\hat{e}_i = \hat{\rho}'x(\hat{\mu}x\hat{\rho}') \tag{5}$$

Inserting equation (4) into equation (3), one finds that

$$J_s = C \sqrt{G_f(\theta',\phi')} \frac{\exp (jk_o\rho')}{\rho'} \left[\hat{n}x(\hat{\rho}'x\hat{e}_i)\right] \tag{6}$$

where $C = 2 \sqrt{2}\left(\frac{\varepsilon_o}{\mu_o}\right)^{\frac{1}{4}}\left(\frac{P_t}{4\pi}\right)^{\frac{1}{2}}$

The integral I then becomes

$$\vec{I} = -j\,\frac{\omega\mu_o}{4\pi R}\;C\,e^{jk_oR}\int_o^{\theta_o}\int_o^{2\pi}\frac{\left[G_f(\theta',\phi')\right]^{\frac{1}{2}}}{\rho'}\exp\left[\,jk_o\rho'+jk_o\vec{\rho}\cdot\hat{R}\,\right]\left[\,\hat{n}x(\hat{\rho}'x\hat{e}_i)\right]r^2\sin\theta\sec\frac{\theta}{2}d\theta d\phi \tag{7}$$

where $\theta_o$ is the limit angle of the reflector dish.

If the reflector is a paraboloid, its surface can be described as

$$x^2 + y^2 = 4f(f-z) \tag{8}$$

The origin of this rectangular coordinate is at the focal point, and

$$\hat{n} = \frac{x\hat{x} + x\hat{x} - 2F\hat{z}}{\sqrt{x^2 + y^2 + 2/f^2}} \tag{9}$$

This $\hat{n}$ is defined in the unprimed or reflector coordinates. However, vector $\hat{\rho}'$ and $\hat{e}_i$ are in the primed coordinates. Furthermore, the integral is performed in terms of the $\theta$ and $\phi$ coordinates, while the $G_f$ function is given in terms of $\theta'$ and $\phi'$. Therefore a means of transforming from the primed coordinate to the unprimed coordinate and a means to convert $\theta'$ to $\theta$ and $\phi'$ to $\phi$ must be provided. In the next section, we shall discuss the coordinate transformation.

## COORDINATE TRANSFORMATION

The coordinate transformation can be performed most conveniently in rectangular coordinates. Let us assume that the origin of the primed coordinates is displaced from the unprimed coordinates by a vector $\vec{\varepsilon}$, then in terms of rectangular coordinates any point specified in the primed coordinate system can be represented in the unprimed coordinate system by the relation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \tag{10}$$

where $\varepsilon_x$, and $\varepsilon_y$ and $\varepsilon_z$ are the components of vector $\vec{\varepsilon}$ defined in the unprimed coordinate.

Besides this translation, the coordinate system may also be rotated. This rotation can be conveniently characterized by three Euler [3,4,5] angles. The transformation has the following relation.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{11}$$

$$\text{where } |A| = \begin{vmatrix} \cos\alpha\cos\gamma-\sin\alpha\sin\beta\sin\gamma & \sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma & -\cos\beta\sin\gamma \\ -\sin\alpha\cos\beta & \cos\alpha\cos\beta & \sin\beta \\ \cos\alpha\sin\gamma+\sin\alpha\sin\beta\cos\gamma & \sin\alpha\sin\gamma-\cos\alpha\sin\beta\cos\gamma & \cos\beta\cos\alpha \end{vmatrix}$$

the three Euler angles $\alpha$, $\beta$ and $\gamma$ are the rotation angles respectively for x, z and y axes. The convention is that a counterclock rotation of the primed coordinate axis into the unprimed coordinate axis is considered to be positive. In conjunction with Eq. (9), a point specified in the primed coordinate system can be converted into an unprimed system by the relation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A] \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \tag{12}$$

A point which is defined in the unprimed coordinate system can be transferred into the primed coordinate by the relation:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [A]^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - [A]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \tag{13}$$

where $[A]^{-1}$ is the inverse of [A] such that

$$[A]\,[A]^{-1} = I$$

One notices that $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$ are defined in terms of unprimed coordinates. By use of the above relation, any point defined in either coordinate system can be transferred to other coordinates, provided the Euler angles and $\vec{\varepsilon}$ are known.

When the integral I of Eq. (7) is calculated, the integration is performed in the unprimed coordinate, however, the primary feed pattern function $G_f(\theta', \phi')$ is defined in the primed coordinates. Furthermore, the distance from primary feed to the reflector surface current point to be integrated ($\rho'$) is also in the primed coordinates. Therefore, for a given point on the reflector surface with known $\theta$, $\phi$ and $\rho$, one must find the corresponding $\theta'$, $\phi'$ and $\rho'$ in the primed coordinates. This can be converted as follows. For a given point with known, $\rho$, $\phi$ and $\phi$, it can be represented in the rectangular coordinate as

$$\vec{p} = \rho\sin\theta\cos\phi\hat{x} + \rho\sin\theta\sin\phi\hat{y} + \rho\cos\theta\hat{z} \tag{14}$$

This can be transformed into primed coordinates by use of Eq. (13), the $\vec{p}$ is then

$$\vec{p} = f_1\hat{x}' + f_2\hat{y}' + f_3\hat{z}' \tag{15}$$

where

$$\begin{vmatrix} f_1 \\ f_2 \\ f_3 \end{vmatrix} = [A]^{-1} \begin{vmatrix} \rho\sin\theta\cos\phi-\varepsilon_x \\ \rho\sin\theta\sin\phi-\varepsilon_y \\ \rho\cos \quad -\varepsilon_z \end{vmatrix}$$

and $\hat{x}'$, $\hat{y}'$ and $\hat{z}'$ are unit vectors in the primed coordinate. Meanwhile this same point can be represented in parameters $\theta'$, $\phi'$, and $\rho'$ as

$$\vec{p} = \rho'\sin\theta'\cos\phi'x' + \rho'\sin\theta'\sin\phi'y' + \rho'\cos\theta'z'$$

Hence one has

$$\rho'\sin\theta'\cos\phi' = f_1(\theta,\phi,\rho) \tag{16a}$$

$$\rho'\sin\theta'\sin\phi' = f_2(\theta,\phi,\rho) \tag{16b}$$

$$\rho'\cos\theta' \quad = f_3(\theta,\phi,\rho) \tag{16c}$$

One can solve for $\theta'$, $\phi'$ and $\rho'$ by use of these three simultaneous equations. The second problem involved in this integral is the cross product of the vectors $\hat{n}$, $\hat{\beta}'$, and $\hat{e}_i$. Both $\hat{\beta}'$ and $\hat{e}_i$ are defined in the prime coordinate, but $\hat{n}$ is defined in the unprimed coordinate. Hence the cross product of $\hat{\beta}'$ and $\hat{e}'$ must be first represented in a rectangular coordinate of the primed system which are then transferred into the unprimed coordinate system by use of Eq. (12). After this transformation is completed one finds

$$\vec{I} = \frac{-j\omega\mu_o}{4\pi R} C \exp(-jk_oR)\int_o^{\theta_o}\int_o^{2\pi} [f_x(\theta,\phi)\hat{x}+f_y(\theta,\phi)\hat{y}+f_z(\theta,c)\hat{z}]$$

$$\bullet \frac{\exp[jk_o\rho'(\theta,\phi)]}{\rho'(\theta,\phi)} \bullet \exp[jk_o\vec{\rho}\cdot\hat{R}]\rho^2\sin\theta\sec\frac{\theta}{2}d\theta d\phi \tag{17}$$

Since the vector $\vec{I}$ is defined in the rectangular coordinate system which has the same coordinates of the field point P, it is straightforward to compute the components $E_\theta$ and $E_\phi$.

Our task is then to evaluate an integrand of the form

$$\vec{I} = k\int_0^{\theta_0}\int_0^{2\pi} f(\theta,\phi)\,\frac{\exp[jk_0\rho'(\theta,\phi)]}{\rho'(\theta,\phi)} \tag{18}$$

$$\exp[jk_0\vec{\rho}\cdot\hat{R}]\rho^2\sin\theta\cos\frac{\theta}{2}\,d\theta d\phi$$

<u>Effect Of Off-set Feed</u>:

If the feed location deviates from the focal point is small, $\rho'$ can be approximated,

$$\rho' = \rho - \hat{\rho}\cdot\vec{\varepsilon}$$

$$\approx \rho - \varepsilon_x\sin\theta\cos\phi - \varepsilon_y\sin\sin\phi - \varepsilon_z\cos\theta \tag{19}$$

Insert this into equation (18), and expand the dot product $\vec{\rho}\cdot\hat{R}$, one finds

$$L = k\int_0^{r_0}\int_0^{2\pi}\frac{f(\theta,\phi)}{\rho'}\exp(jL)\,rd\theta d\phi \tag{20}$$

where $r = \rho\sin\theta$

$$L = -k_0(\rho\cos\theta\cos\Theta + \rho + \varepsilon_z\cos\theta) + k_0 r\sin\Theta\cos\phi\cos\Phi$$

$$+k_0 r\sin\Theta\sin\phi\sin\Phi + k_0\varepsilon_x\sin\theta\cos\phi + k_0\varepsilon_y\sin\theta\sin\phi \tag{21}$$

For a paraboloid,

$$\rho = \frac{2f}{1-\cos\theta} \tag{21a}$$

$$\sin\theta = \frac{4fr}{4f^2+r^2} \tag{21b}$$

$$\cos\theta = \frac{4f^2-r^2}{4f^2+r^2} \tag{21c}$$

Equation (21) can be manipulated into the following form.

$$L = -2fk_o - k_o\rho\cos\theta(1-\cos\theta) + k_o r\sin\theta\cos\phi\cos\Phi + k_o r\sin\theta\sin\phi\sin\Phi$$
$$+k_o r\varepsilon'_x\cos\phi + k_o r\varepsilon'_y\sin\phi + k_o r\varepsilon'_x\cos\theta\cos\phi + k_o r\varepsilon'_y\cos\theta\sin\phi$$
$$+k_o\varepsilon'_z\, 2f\cos\theta \qquad (22)$$

$$\varepsilon'_x = \varepsilon_{x/2f} \qquad (23a)$$

$$\varepsilon'_y = \varepsilon_{y/2f} \qquad (23b)$$

where
$$\varepsilon'_z = \varepsilon_{z/2f} \qquad (23c)$$

Let
$$\sin\theta'\cos\Phi' = \sin\theta\ \cos\Phi + \varepsilon'_x \qquad (24a)$$
$$\sin\theta'\sin\Phi' = \sin\theta\ \sin\Phi + \varepsilon'_y \qquad (24b)$$

These two equations represent the effect of off-focal point field which shifts the radiation pattern from a point $\theta$ and $\Phi$ to a point $\theta'$ and $\Phi'$. Set

$$L_a = \rho\cos\theta(\cos\theta - 1) \qquad (25a)$$
$$M = \sqrt{\varepsilon_x^2 + \varepsilon_y^2} \qquad (25b)$$
$$\eta = \tan^{-1}\ \varepsilon'_y/\varepsilon'_x \qquad (25c)$$

$$L_c = r\,\frac{1-\left(\frac{r}{2f}\right)^2}{1+\left(\frac{r}{2f}\right)^2}\left(M\cos(\phi-\eta)-\varepsilon'_z\ \frac{2f}{r}\right) \qquad (25d)$$

Insert these relations into equation (2), one finds

$$I = k\int_0^{r_o}\int_0^{2\pi} F(r,\phi)\exp[jk_oL_a]\exp[jk_o r\sin\theta'\cos(\phi-\Phi')]\ r\,dr\,d\phi \qquad (26)$$

Where
$$F(r,\phi) = \frac{f(r,\phi)}{\rho'}\ \exp[-j2k_of]\exp[-jk_oL_c] \qquad (27)$$

If the primary feed is located at the focal point, then $\varepsilon_x$, $\varepsilon_y$, and $\varepsilon_z$ are all zero. Under this condition, $\theta' = \theta$, $\Phi' = \Phi$ and $L_c = 0$. Clearly the effect of off-focal point feed not only introduces additional phase delay, it also changes the beam point direction.

Integration of equation (26) involves both $\theta, \Phi$ and $\Theta$, $\Phi$ variables. This appears in two places. The first place is the $\exp[+jk_oL_a]$ term. For this term one may calculate this integration at a given $\theta$ angle for all points at the vicinity of this angle. Larger errors will certainly appear at points which are remote from this angle. However, there is a way to correct this error. This will be discussed later. At the time being, we shall assume that $L_a$ is computer at a given $\Theta$ and it is only a function of $\theta$.

The second place is the $\exp[jk_o r\sin\theta'\cos(\phi-\Phi')]$ term which we shall show in the next section that it can be integrated into a closed form, if $F(r,\phi)$ is expanded into an appropriate orthogonal series.

One notices that the integral of equation (26) involves variable $\theta$ and $\Phi$. Thus for each field point computed, this integration must be repeated all over again. For simulations of a reflector antenna or computing the radiating pattern, it usually involves a large number of field points. The required computer time is astronomical. To overcome this difficulty, Galindo-Israel and Mittra [6] proposed to expand $F(\theta,\phi)$ into an orthogonal series so that equation (27) can be integrated into closed forms as a function of $\Theta$ and $\Phi$ of the field coordinates. Since the series is orthogonal, the coefficient of the expanded series can be easily evaluated. Galindo-Israel and Mittra showed that this series converges very fast, only a few terms are needed. This will greatly reduce the required computer time. This procedure will be summarized as follows:

Let's set

$$r = sr_o$$

where $r_o$ is the size of the reflector dish. Equation (27) can be written

$$I = kr_o^2\int_o^1\int_o^{2\pi} F(s,\phi)\exp[jk_oL_a + jk_o r\sin\Theta'(\phi-\Phi')]\, sdsd\phi \quad (28)$$

Let

$$F(s,\phi)\exp[jk_oL_a] = \sum_{n,m} (C_{nm}\cos n\phi + D_{nm}\sin n\cdot\phi)P_m^n(1-2s^2) \quad (29)$$

where $P_m^n(1-2s^2)$ is a modified Jacobi polynomial which relates to the conventional Jacobi polynomial by the following relationship

$$P_m^n(1-2s^2) = \sqrt{2(n+2m+1)}\; P_m^{(n,o)}(1-2s^2)s^n. \quad (30)$$

The modified Jacobi polynomials have the orthogonal property, that

$$\int_o^1 P_m^n (1-2s) \; P_{m'}^n (1-2s^2) sds = \delta_{mm'} \tag{31}$$

Where $\delta_{mm'}$ is the delta function. The Jacobi polynomials form a complete set similar to that of a Fourier series which also forms a complete set. These functions can be used to represent any arbitrary function if proper coefficients are chosen. Insert this relation into equation (28), one has

$$I = kr_o^2 \int_o^1 \int_o^{2\pi} (\sum_{n,m} C_{n,m} (\cos n\phi + D_{nm} \sin n\phi) \tag{32}$$

$$\cdot \exp[jk_o r_o s \sin\theta' \cos(\phi-\Phi')] d\phi \cdot P_m^n (1-2s^2) \; sds$$

By use of the identity,

$$\int_o^{2\pi} \cos n\mu e^{-jZ\cos\mu} d\mu = \frac{2\pi}{j^n} J_n(Z) \tag{33}$$

Equation (32) becomes

$$I = kr_o^2 \int_o^1 \sum_{n,m} \frac{2\pi}{j^n} (C_{nm} \cos n\Phi + D_{nm} \sin n\Phi) \; J_n(k_o r_o s \sin\theta')$$

$$\cdot p_m^n (1-2s^2) sds \tag{34}$$

By use of the identity

$$a \int_o^1 s^{n+1/2} \; P_m^{(n,o)} (1-2s^2) \; J_n(yas)(ys)^{1/2} \; ds$$

$$= \frac{1}{y^{1/2}} J_{n+2m+1}(ay) \qquad n > -(m+1) \tag{35}$$

Equation (34) becomes

$$I = kr_o^2 \; 2\pi \sum_n \sum_m \frac{1}{j^n} \sqrt{2(n+2m+1)} \; (C_{mn} \cos n\Phi + D_m \sin n\Phi)$$

$$\cdot \frac{J_{n+2m+1}(k_o r_o \sin\theta')}{k_o r_o \sin\theta'} \tag{36}$$

The $C_{mn}$ and $D_{mn}$ are evaluated as follows:

$$\begin{bmatrix} C_{mn} \\ \\ D_{mn} \end{bmatrix} = \int_0^1 \int_0^{2\pi} F(s,\phi) \exp[jk_o L_a] \begin{bmatrix} \cos n\phi \\ \\ \sin n\phi \end{bmatrix} d\phi \tag{37}$$

$$\cdot \; P_m^n \; (1-2s^2) s ds$$

In this formulation, the integrand I is evaluated at a certain $\theta$ angle. For field points at the vicinity of this angle, this integral gives a very good approximation. However, for points which are remote from this angle, the phase error becomes intolerable. To reduce this error, the expotential term of Equation (37) may be expanded into a Taylor series. When $C_{mn}$ and $D_{mn}$ are evaluated, the integration can be carried out term by term. High order terms can be calculated by use of an iterative process. Hence, the integration needs to be performed only once. This procedure will be shown as follows.

For a parabloid,

$$\rho\cos\theta = \frac{r_o^{\,2}}{4f}\left(\frac{4f^2}{r_o^{\,2}} - s^2\right) \tag{38}$$

Hence

$$\exp[jk_o L_a] = \exp\left[ jk_o (1-\cos\theta) \frac{r_o}{4f} \left( s^2 - \frac{4f^2}{r_o^2} \right) \right]$$

$$= 1 - jH(s^2 - s_a^2) + \frac{(jH)^2}{2'} (s^2 - s_a^2)^2 \cdots \tag{39}$$

where $H = k_o \dfrac{r_o}{4f} (1-\cos\theta)$

$S_a = \dfrac{2f}{r_o}$

The coefficient $C_{mn}$ and $D_{mn}$ takes the following form

$$\begin{matrix} C_{mn} \\ D_{mn} \end{matrix} = \int_o^1 G(s,m,n) P_m^n (1-2s^2) sds - jH \int_o^1 G(s,m,n) P_m^n (1-2s^2) sds$$

$$+ \frac{(jH)^2}{2!} \int_o^1 G(s,m,n) P_m^n (1-2s^2) sds \; ..... \qquad (40)$$

where

$$G(s,m,n) = \int_o^1 F(s,\phi) \begin{matrix} \cos n\phi \\ \sin n\phi \end{matrix} d\phi$$

Let $\quad (B_{mn})^o = \int_o^1 G(s,m,n) P_m^n (1-2s^2) sds \qquad (41)$

Then $\quad (B_{mn})^1 = \int_o^1 G(s,m,n)(s^2-s_a^2) P_m^n (1-2s^2) sds \qquad (42)$

From Magnus [7], one has

$$(s^2-s_a^2) P_m^n = a_{mn} P_{m-1}^n + (b_{mn}-s_a^2) P_m^n \qquad (43)$$

$$+ C_{mn} P_{m+1}^n$$

where

$$a_{mn} = - \frac{m(n+m)}{(n+2m)(n+2m+1)} \qquad (44)$$

$$b_{mn} = - \frac{(m+n)^2}{(2m+n)(n+2m+1)} \qquad (44b)$$

$$C_{mn} = - \frac{(m+1)(n+m+1)}{(n+2m+1)(n+2m+2)} \qquad (44c)$$

Therefore

$$(B_{mn})^{1} = a_{mn}(B_{m-1,n})^{0} + (b_{mn} - s_{a}^{2}\ (B_{mn})^{0}$$

$$+ c_{mn}(B_{m+1,n})^{0} \tag{45}$$

and

$$(B_{mn})^{p} = a_{mn}(B_{m-1,n})^{p-1} + (b_{mn} - s_{a}^{2})\ (B_{mn})^{p-1}$$

$$+ c_{mn}(B_{mn+1,n})p-1 \tag{46}$$

It is evident we may use this iteration process to compute all higher order $B_{mn}$'s. Hence, this correction can be carried out to any desired degree.

APPROXIMATE SOLUTION

Note in Equation (36), that the leading term (m=0,n=0) is $\dfrac{J_1(k_o r_o \sin\theta')}{k_o r_o \sin\theta'}$ the pattern of an undistorted uniformly excited equivalent circular aperture. All the remaining terms contribute zero at $\sin\theta'=0$. The higher order terms are thus perturbation of a well-shaped leading term beam. This term therefore can be used to approximate the radiation pattern at the vicinity of the main beam. Since off-focal point feed shifts the beam point direction, we shall find this angle deviation.

Let's assume that

$$\theta' = \theta + \Delta\theta \tag{47a}$$

$$\Phi' = \Phi + \Delta\Phi \tag{47b}$$

If $\Delta\theta$ and $\Delta\Phi$ are small, equation (24a) and (24b) can be approximated by the following relations:

$$\Delta\theta\cos\theta\cos\Phi - \Delta\Phi\sin\theta\sin\Phi = \varepsilon_x/2f \tag{48a}$$

$$\Delta\theta\cos\theta\sin\Phi + \Delta\Phi\sin\theta\sin\Phi = {}_{y}/2f \tag{48b}$$

Solving $\Delta\theta$ and $\Delta\Phi$ from these two simultaneous equations one finds

$$\Delta\theta = \frac{\cos\Phi\ \varepsilon_x/2f + \sin\Phi\ \varepsilon_y/2f}{\cos\theta} \tag{49a}$$

$$\Delta\Phi = \frac{\cos\Phi\ \varepsilon_x/2f - \sin\Phi\ \varepsilon_y/2f}{\sin\theta} \tag{49b}$$

These two equations can be used to locate the shifted antenna pattern.

## COMPUTER PROGRAM

A computer program which computes the radiation pattern of a reflector disk using the approach described in this report is compiled in FORTRAN language and has been run on an ASC machine at NRL. Five input data cards to this program are required to specify the parameters of the dish to be computed. They are listed as follows:

1. The first card specifies the axis rotation angles of the coordinate of the feed to the coordinate of the reflector.

2. The second card specifies the sequence of these coordinate rotation described in 1.

3. The third card specifies the displacement of the feed from the reflector focal point.

4. The fourth card specifies the number of terms of the orthogonal series to be used.

5. The fifth card defines the size of the disk and the distance of the focal point and also specifies the feed polarization.

Figure 2 shows an example of such a computed pattern. This figure shows respectively x, y and z components of the radiation of a dish of a size, $R = 12.5\lambda$ and $f = 12.5\lambda$. The feed horn is linearly polarized at Y - direction and has a taper of 10 dB. Location of this feed is displaced from the focal point in x direction by an amount of $\varepsilon_x = 3.18\lambda$. $KAU = 2 \times \pi \times R \times \sin\theta$. Where $\theta$ is the angle of a field point. The plot is at a cut of $\Phi = 0$.

To achieve this plot a total of 200 field points has been computed. Total computer time of this plot including compiling and evaluating coefficient is 41 seconds on an ASC machine.

## REFERENCES


1. "Microwave Antenna Theory and Design," Edited by Samuel Silver, MIT Radiation Laboratory, Series Vol. 12, 1949.

2. "Antenna Theory", edited by R.E. Collin and I.J. Zucker, McGraw-Hill, 1969.

3. "Classical Mechanics", Herbert Goldstein, Addison-Wesley, 1950.

4. "Computer Analysis of Conformal Phased Arrays", J.K. Hsiao and J.B.L. Rao, NRL Report 7826, Nov. 15, 1974.

5. "Computation of Far-Field Pattern and Polarization of Conformal Arrays", J.K. Hsiao and J.B.L. Rao, IEEE Trans. on Antennas and Propagation, March 1976.

6. "A New Series Representation for the Radiation Integral with Application to Reflector Antenna", Victor Golindo-Israel and Raj Mittra, IEEE Trans. on Antennas and Propagation, Vol AP-25, No. 5, September 1977.

7. W. Magnus, F. Oberhettinger, and R.P. Soni, "Formulas and Theorems for the Special Functions of Mathematical Physics", Springer-Veslag, 1966.

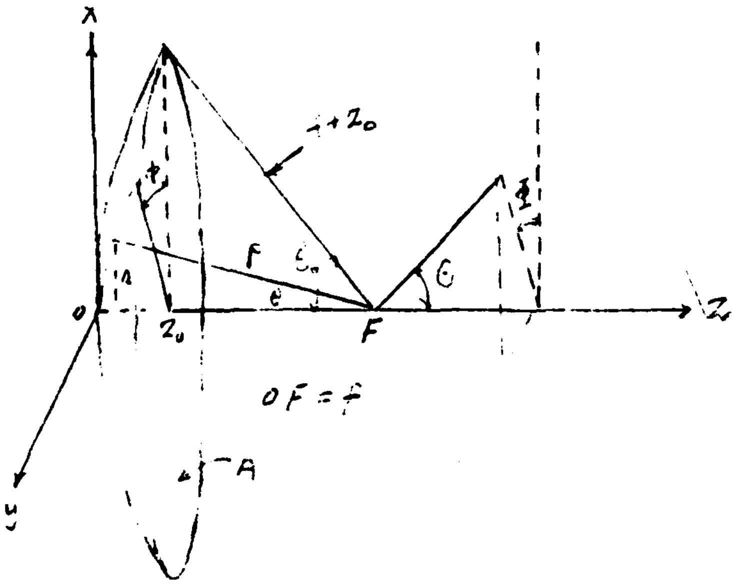


Fig. 1 — Coordinate systems of a reflector antenna

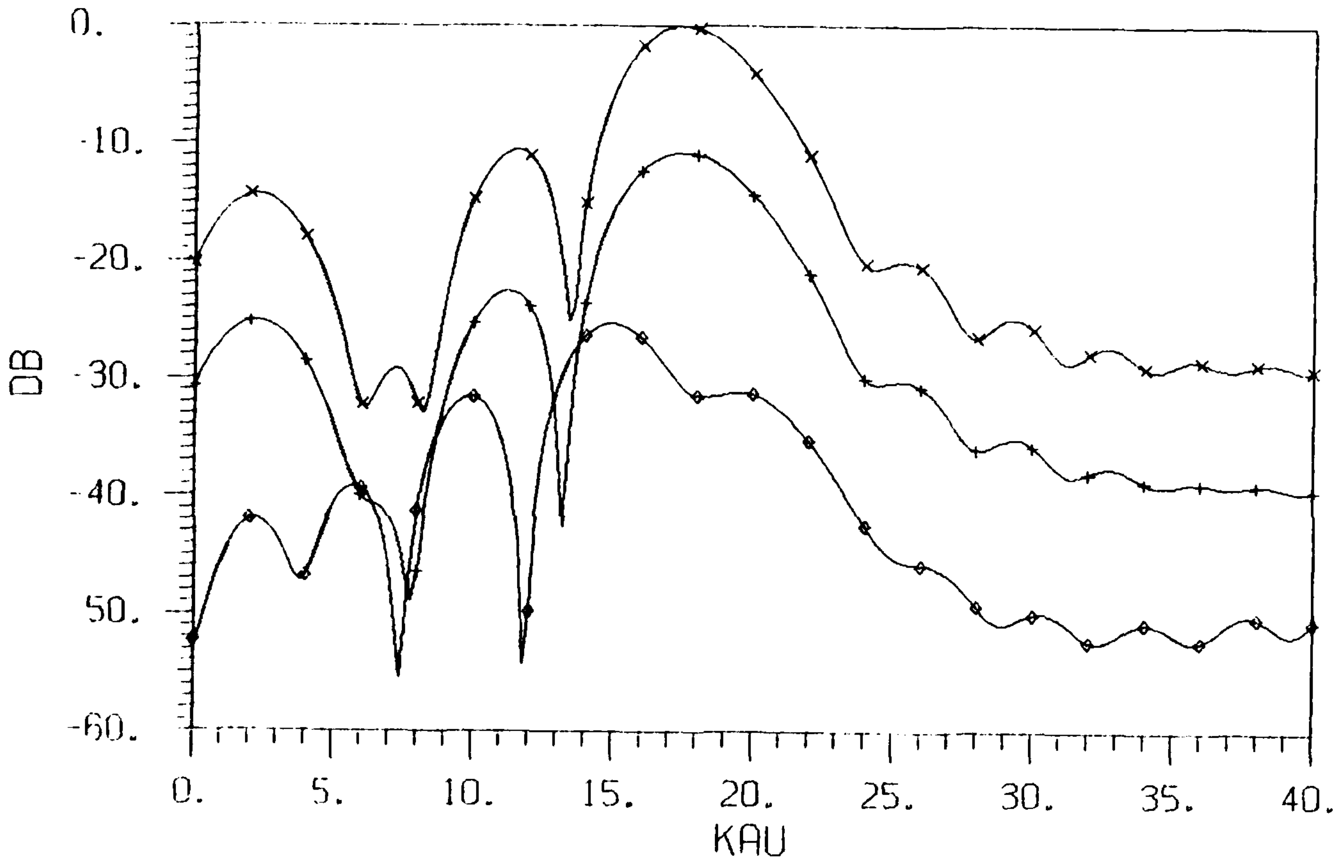


**Fig. 2 — Radiation pattern of x, y and z components of a reflector dish, R = 12.5λ, $f_o$ = 12.5λ. Feed is displaced, $\epsilon_x$ = -3.18λ.**

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